OPERATIONS RESEARCH
Vol. 53, No. 5, September–October 2005, pp. 745–763
ISSN 0030-364X | EISSN 1526-5463 | 05 | 5305 | 0745

DOI 10.1287/opre.1050.0231

OR PRACTICE

A Two-Sided Optimization for Theater Ballistic Missile Defense

Gerald Brown, Matthew Carlyle, Douglas Diehl, Jeffrey Kline, Kevin Wood
Operations Research Department, Naval Postgraduate School, Monterey, California 93943
{gbrown@nps.edu, mcarlyle@nps.edu, ddiehl@nps.edu, jekline@nps.edu, kwood@nps.edu}

We describe JOINT DEFENDER, a new two-sided optimization model for planning the pre-positioning of defensive missile interceptors to counter an attack threat. In our basic model, a defender pre-positions ballistic missile defense platforms to minimize the worst-case damage an attacker can achieve; we assume that the attacker will be aware of defensive pre-positioning decisions, and that both sides have complete information as to target values, attacking-missile launch sites, weapon system capabilities, etc. Other model variants investigate the value of secrecy by restricting the attacker's and/or defender's access to information. For a realistic scenario, we can evaluate a completely transparent exchange in a few minutes on a laptop computer, and can plan near-optimal secret defenses in seconds. JOINT DEFENDER's mathematical foundation and its computational efficiency complement current missile-defense planning tools that use heuristics or supercomputing. The model can also provide unique insight into the value of secrecy and deception to either side. We demonstrate with two hypothetical North Korean scenarios.

Subject classifications: missile defense; optimization; bilevel integer linear program; mixed-integer linear program.
Area of review: OR Practice.
History: Received June 2004; revision received September 2004; accepted March 2005.

They may vex us with shot, or with assault. To intercept this inconvenience, a piece of ordnance 'gainst it I have placed.
Shakespeare, *Henry IV*

1. Theater Ballistic Missile Defense: Background

This paper introduces JOINT DEFENDER, a new, bilevel (i.e., two-sided) optimization model to help plan the pre-positioning of the defensive interceptor platforms that the United States and its allies are deploying to counter exigent theater ballistic missile threats. Solutions require only a few seconds or minutes on a personal computer and can yield important new insights.

1.1. The Theater Ballistic Missile Threat

Theater ballistic missiles (TBMs) can deliver high-explosive, chemical, biological, or nuclear warheads over long distances. Although no potential adversary other than Russia possesses TBMs capable of striking the United States, both China and North Korea are developing missiles that will likely have that capability by 2015 (CIA 2001). Existing short-range and medium-range TBMs pose immediate threats in many regional conflicts, however, as demonstrated in the first and second Gulf wars. Figure 1 illustrates some TBMs that currently concern military planners.

North Korea is particularly worrisome. It is known to be developing and exporting ballistic missiles and missile technology, and has numerous indigenous missile-production facilities and prepared launch sites. Figure 2 depicts some of those launch sites and the areas they threaten.

North Korea is developing longer-range intercontinental ballistic missiles (e.g., the Taep'o-Dong II) that will be capable of striking the western coast of the United States and Alaska (CIA 2001). Given that North Korea also claims to have developed fission weapons, it is vital that we understand how to best deploy (i.e., pre-position) interceptor platforms to defend against TBM attacks from that country.

In response to such threats, this paper develops JOINT DEFENDER, a bilevel integer linear program for pre-positioning theater ballistic missile defense (TBMD) assets, and demonstrates how to analyze scenarios using two hypothetical Korean examples. Before developing this new model, we first describe the interceptor platforms that have been fielded or are under development, and review the analytical tools currently used to plan deployment of these platforms.

1.2. TBM Interceptor Platforms

Figure 3 shows three components of the United States joint missile defense, which we will use as representative defensive platforms.

The Army's PATRIOT anti-missile missile system is currently deployed and has been used in Operation Iraqi Freedom. PATRIOT provides terminal defense against ballistic missiles, cruise missiles, and aircraft. It consists of

Report Documentation Page

Form Approved
OMB No. 0704-0188

1. REPORT DATE **OCT 2005**	2. REPORT TYPE **N/A**	3. DATES COVERED **-**

4. TITLE AND SUBTITLE **A Two-Sided Optimization for Theater Ballistic Missile Defense**	5a. CONTRACT NUMBER
	5b. GRANT NUMBER
	5c. PROGRAM ELEMENT NUMBER
6. AUTHOR(S)	5d. PROJECT NUMBER
	5e. TASK NUMBER
	5f. WORK UNIT NUMBER
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) **Naval Postgraduate School Operations Research Department Monterey, CA 93943**	8. PERFORMING ORGANIZATION REPORT NUMBER
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)	10. SPONSOR/MONITOR'S ACRONYM(S)
	11. SPONSOR/MONITOR'S REPORT NUMBER(S)

12. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release, distribution unlimited

13. SUPPLEMENTARY NOTES

14. ABSTRACT

15. SUBJECT TERMS

16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
a. REPORT **unclassified**	b. ABSTRACT **unclassified**	c. THIS PAGE **unclassified**	**SAR**	**19**	

Standard Form 298 (Rev. 8-98)
Prescribed by ANSI Std Z39-18

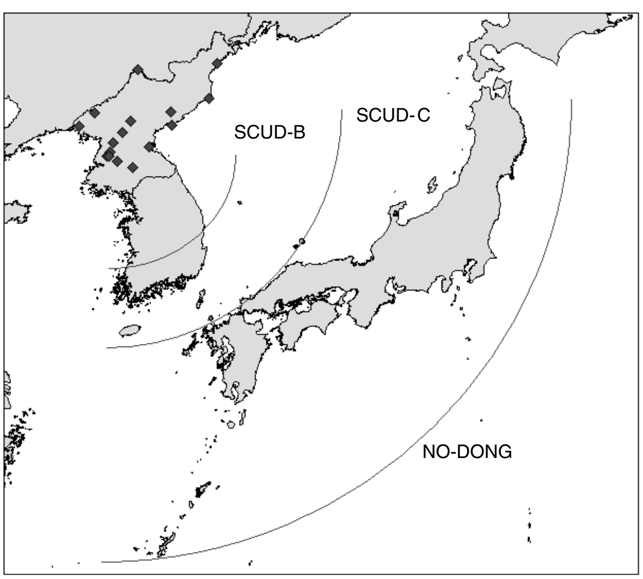
Figure 1. Current ballistic missile threats. Shown left to right are a North Korean Scud-B transporter-erector-launcher (TEL), a TEL firing a missile, and an Iranian fixed ballistic missile launcher.

a mobile launcher, a phased-array air search-and-tracking radar, plus various command and support vehicles. It can shoot three types of interceptor missiles, the PAC-2, PAC-2 GEM, and PAC-3 (Jane's 2003c).

The Army is developing theater high altitude air defense (THAAD), which will provide a midcourse, high-altitude defense against ballistic missiles using a kinetic-kill interceptor. THAAD's physical composition resembles that of PATRIOT (Jane's 2003c).

"Navy AEGIS" refers to deployed Ticonderoga-class guided-missile cruisers and Arleigh Burke-class guided-missile destroyers. Each of these ships carries the AEGIS SPY-1 phased-array radar and can function as a TBM interceptor platform. These ships currently carry Standard Missile-2 (SM2) variants that provide terminal defense against cruise missiles and aircraft. The Navy is now developing the Standard Missile-3 (SM3), a kinetic-kill exoatmospheric interceptor, to provide a midcourse defense against TBMs (Jane's 2003b).

Figure 2. Approximate maximum ranges of North Korean Scud-B, Scud-C, and No-Dong theater ballistic missiles. Note that all of Japan and Okinawa are vulnerable to the No-Dong missile.



TBMD has become an important component of the Department of Defense research and development budget (Department of Defense 2004), and we may expect the United States to field a number of new TBMD systems in the next few years. Indeed, an air-based laser is already under development (Jane's 2003e). We do not include this future system in our demonstration scenarios, but incorporating such innovations in JOINT DEFENDER is straightforward.

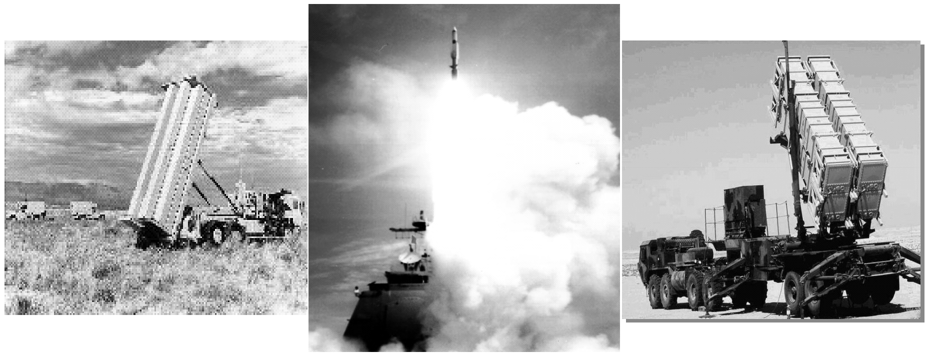
1.3. Current TBMD Planning Tools

Effective pre-positioning of TBMD assets is crucial given that (a) a defensive interceptor has limited range, (b) it can destroy a TBM only at certain points in the TBM's trajectory, and (c) that trajectory will depend on the type of TBM and its launch and target coordinates. Currently, joint forces commanders can plan pre-positioning using several analytical tools; we describe these next and point out their strengths and weaknesses.

Area Air Defense Commander System. The Navy's area air defense commander system (AADCS), AN/UYQ-89, is currently deployed on command ships USS BLUE-RIDGE, USS MOUNT WHITNEY, the AEGIS cruiser USS SHILOH, and at the Joint National Integration Center in Colorado (Jane's 2003a). In addition to modules for real-time tracking of assets and threats, AADCS contains a planning module that enables air-defense commanders to plan and "war-game" potential TBMD scenarios.

AADCS uses 32-processor Silicon Graphics supercomputers to evaluate, using an enumeration-based myopic heuristic, a sequence of increasingly complex "defense plans" before committing to a good one (Silicon Graphics Incorporated 2003). For each target in a scenario, in priority order, AADCS enumerates every possible combination of (a) enemy launch site, (b) missile type, (c) interceptor-platform position on a fine geographical grid, and (d) intercept salvo (set of interceptors that might be shot at the TBM). For each of those combinations, it evaluates the probability of intercepting the TBM successfully. Once AADCS identifies the required platform(s), position(s),

Figure 3. TBMD platforms, deployable or under development. Shown left to right are a THAAD launch vehicle, an AEGIS guided missile cruiser firing a standard missile, and a PATRIOT launch vehicle.



and corresponding salvo(s) that yield a sufficiently high probability of intercept, the next target in the priority list is analyzed, more platforms are positioned, and so on. (Once a platform's position is fixed, it is also considered for defending subsequent, lower-priority targets.) AADCS provides an estimate of defense coverage and an expected number of enemy missiles that will leak through the defense plan. AADCS's brute-force enumeration determines an optimal defense for its highest-priority target, and that enumeration goes on to optimize a sequence of restricted problems. Several weaknesses are apparent: the procedure is essentially a sequential greedy heuristic with no guarantee of global optimality, it ignores the enemy's strategy, and it requires an expensive computational platform.

Theater Battle Management Core Systems (TBMCS). The U.S. Air Force's air operations centers use theater battle management core systems (TBMCS) for theater-level planning in support of the area air defense commander. TBMCS supports strategic planning, air battle planning, and mission preparation, together with mission execution, reporting, and analysis; the last items are supported in near real-time as situations unfold. A module in TBMCS automates an overlay of potential "launch fans" by defensive "interceptor envelopes." That is, the module evaluates a manually-prepared, pre-positioning plan for defenses by analyzing the intersection of (a) the two-dimensional projection on the earth's surface of the three-dimensional region that might be traversed by a TBM, and (b) a similar projection for the "kill zone" of an interceptor shot from a given position. This procedure suggests a plausible solution that indicates whether or not a hypothesized attacking missile can be struck by a pre-positioned interceptor, subject to the error induced by two-dimensional projections. TBMCS cannot optimize defense plans because it requires human intervention (i.e., guessing), it does not measure expected damage incurred by an attack, and it ignores the enemy's strategy.

Commander's Analysis and Planning Simulation. Since 1993, the Missile Defense Agency has sponsored commander's analysis and planning simulation (CAPS), which is currently hosted by theater ballistic missile-planning cells of Central Command, European Command, Pacific Command, Strategic Command, the Naval Postgraduate School, and others—a total of more than 50 sites.

CAPS helps assess defense-system capabilities and positioning. The performance of manually-prepared defense plans can be evaluated against manually-prepared threat scenarios (Sparta 2004). The CAPS operator selects the "best-looking" defense plan that appears to protect defended assets (targets) with high probability and appears to maximize the number of missiles the defender can engage. CAPS does not make the two-dimensional approximations that TBMCS does, but it still requires human intervention and ignores the enemy's strategy.

All three fielded systems, AADCS, TBMCS, and CAPS, address the complex problem of TBM defense in very different ways, with wide variation in computational requirements, degrees of fidelity, and objectives. These systems can be used to search for "good" defense plans, but only through manual or automated heuristics. None of the systems account for how the enemy might change his strategy in response to observing pre-positioned TBMD assets, i.e., in response to observing an implemented defense plan.

2. A New Two-Sided Optimization for TBMD Planning

We introduce a new paradigm for planning the pre-positioning of TBMD assets. We first express enemy courses of action as an "inner" mathematical optimization that maximizes expected damage subject to known defensive positions and capabilities. An "outer" optimization minimizes that maximum by pre-positioning defensive platforms and committing to intercept strategies appropriately.

We can most conveniently express our model for TBMD as a bilevel integer linear program (BLILP) (e.g., Moore and Bard 1990). Then, because of its special structure, we can convert our BLILP into a standard mixed-integer linear program (MILP) to actually solve it. With the roles of attacker and defender reversed, this general idea has been successfully used to model a number of network-interdiction problems (Phillips 1993, Wood 1993, Israeli and Wood 2002; see Whiteman 2000 for details on an

application; see Fulkerson and Harding 1977, Golden 1978 for earlier, bilevel linear-programming models involving continuous interdiction effort; and see Salmeron et al. 2004 for an application of a bilevel optimization to interdicting electric power grids). In these network-interdiction problems, an interdictor uses limited offensive resources to attack and damage an adversary's network (e.g., road system, communications network) to minimize the maximum benefit his adversary can obtain from it. Skroch (2004) and Brown et al. (2004) model the optimal disruption of a weapons development program by interdicting a project network. Their BLILP cannot be converted to a MILP, and is solved with a decomposition algorithm.

2.1. TBMD Terminology and Assumptions

The following terminology and assumptions characterize JOINT DEFENDER.

Both sides have full knowledge of the parameters and data described below.

Each *launch site* for attacking missiles is located by latitude and longitude. Any finite number of dispersed launch sites may exist.

Each *target* vulnerable to enemy attack is located by latitude and longitude. Any finite number of dispersed targets may exist, and each has a *target value*.

Each *candidate defender position* is located by latitude and longitude. Any finite number of candidate positions may exist. The set of positions can include, for example, a discretized field of grid points with desired geographic fidelity.

Each enemy *missile* has a minimum and maximum range, and can hit any target within this range of its launch site with a known *probability of kill*. This probability can depend on the *missile type*, target, and range from launch site to target.

An enemy *attack* consists of a launch of a missile from an enemy launch site against a vulnerable target. The enemy's goal is to launch a set of attacks that maximizes total expected target damage.

Each defender *class* consists of a given number of individual *platforms*, each endowed with a *loadout* of a given number of each type of *interceptor* weapon (anti-missile missiles are the only extant interceptors, but other types, like the air-based laser, are under development). Each defending platform may be located at any candidate defender position that is secure and compatible for its class. That is, ships may only be positioned at sea, land units on compatible terrain, and air defenders in safe airspace.

A single attack (one missile from one launch site to one target) may be engaged by any defending platform with an *intercept salvo* of any number of any types of interceptor missiles available on that platform. For planning purposes, and as a matter of effective tactical doctrine, we assume that the planned intercept of each enemy missile will be executed by a single defending platform. (In execution, this would not preclude defending platforms from providing a layered defense to defended targets, but we do not address this complication.)

The *probability of negation* defines the probability that an intercept salvo will destroy the attacking missile; this varies by attack launch site, missile type, target location, defender position, defending salvo, and any synergy among the intercepting missiles in that salvo. The geometry of such an engagement can be depicted by an oblate spherical triangle, with apexes at the launch site, the target location, and the defender's position. The probability of negation for an intercept salvo is then a function of (a) the relative positions of missile and interceptor, (b) the vulnerability of the attacking missile to the interceptor at the point of intercept—some interceptors can only strike a missile traversing its early- or middle-phase *flyout trajectory*, and some provide only *terminal-phase* defense—and (c) the combined effectiveness of the entire intercept salvo. In practice, JOINT DEFENDER uses probabilities of negation computed through a mathematical approximation, or through lookup and interpolation of engineering estimates in "cross-range" and "down-range" tables for each type of intercept salvo and missile altitude. The probability of negation for an interceptor salvo *does not rely on an independence assumption* among missiles in that salvo.

2.2. Mathematical Development of JOINT DEFENDER

The attacker controls a set of launch sites $s \in S$, and possesses $\overline{fixed}_{m,s}$ missiles of type $m \in M$ pre-positioned at site s, as well as a pool of $\overline{mobile}_m$ missiles that can be transported to any capable receiving launch site. Transport of the mobile missiles may be restricted by $\underline{move}_{m,s}$ and/or $\overline{move}_{m,s}$. Launch site s can launch no more than $\overline{fixed}_{m,s} + \overline{move}_{m,s}$ missiles of type m. (Of course, if the defender knows that launch site s is incapable of launching missile type m, $\overline{fixed}_{m,s} = \overline{move}_{m,s} = 0$.) The defender guards a set of targets $t \in T$, with each target t having value val_t. An attack $a \in A$ consists of a launch from site $s_a \in S$ of a missile of type $m_a \in M$ at a target $t_a \in T$. This attack will hit and destroy the target with probability of kill Pk_a, assuming that the defender takes no action. An upper bound $\overline{missiles}_t$ may be placed on the number of missiles the attacker will launch at target t. The attacker must decide which missiles to launch at which targets to maximize total expected target damage, weighted by target value.

The defender controls a set of defending platforms $p \in P$, each of which is a member of platform class $c_p \in C$. Each platform of class c can be pre-positioned at any one location $g \in G_c \subseteq G$. Each platform p carries $loadout_{p,i}$ defensive interceptors of type $i \in I$. An attack a can be engaged with alternative defensive actions $d \in D$, where defense d launches $salvo_{a,c,d,i}$ interceptors of type(s) i and succeeds in thwarting the attack with probability of negation $\mathrm{Pn}_{a,c,g,d}$. Each defensive engagement is conditional, meaning that if attack a is not launched, then no interceptor devoted to engaging that attack is launched.

The defender wishes to optimize defensive prepositioning for attack interception *while assuming the attacker will observe these preparations and optimize his (multimissile) attack to exploit any weaknesses in these defenses*. The defender's objective is to minimize the maximum total expected damage to targets. We note that this model is a conservative one for the defender because he must protect against the worst possible attack. It is conservative for the attacker, because he must plan against the best possible defense. However, variants of the model we describe later enable analysis of a range of situations, from conservative to optimistic, for either opponent.

Model JD-MINMAX: Minimize Maximum Expected Total Damage

Indices and Index Sets

Attacker

- $m \in M$ attacking missile types
- $s \in S$ attacker launch sites
- $t \in T$ targets ("defended asset")
- $a \in A$ attacks (a single missile launched at a target)
- $a \in A_{m,s} \subseteq A$ attacks launching a missile of type m from site s
- $a \in A_t \subseteq A$ attacks a with target t
- s_a launch site of attack a, $s_a \in S$
- m_a missile type launched in attack a, $m_a \in M$
- t_a target of attack a, $t_a \in T$

Defender

- $p \in P$ defending platforms
- $c \in C$ defending platform classes
- c_p class of platform p, $c_p \in C$
- $g \in G$ candidate stationing positions for a defending platform
- $g \in G_c \subseteq G$ candidate stationing positions for a defending platform of class c
- $i \in I$ defensive interceptor types
- $d \in D$ defense options

Data (units)

Attacker

- $\overline{mobile}_m$ attacker's total supply of mobile missile type m (missiles)
- $\overline{fixed}_{m,s}$ attacker's total supply of stationary type m missiles at launch site s (missiles)
- $\underline{move}_{m,s}$, $\overline{move}_{m,s}$ minimum and maximum number of mobile missile type m that attacker can transport to launch site s (missiles)
- $\overline{missiles}_t$ maximum number of missiles that can attack target t (missiles)
- val_t value of target t (value)
- Pk_a probability that attack a hits and destroys its target t_a if not intercepted, i.e., probability of kill (fraction)

Defender

- $loadout_{p,i}$ number of type i interceptors carried by platform p (interceptors)
- $salvo_{a,c,d,i}$ number of type i interceptors used against attack a by a class c platform exercising defense option d (interceptors)
- $\overline{shoot}_p$ maximum number of interceptors platform p can shoot in an exchange (interceptors)
- $\mathrm{Pn}_{a,c,g,d}$ probability that attack a is negated if platform p, class $c = c_p$, in position $g \in G_{c_p}$ exercises defense option d, i.e., probability of negation (fraction)

Variables (units)

Attacker

- $W_{m,s}$ number of type m mobile missiles transported to launch site s (missiles)
- Y_a 1 if attack a is launched, 0 otherwise (binary) (**Y**, the vector of attacks by individual missiles, is an "attack plan")

Defender

- $X_{p,g}$ 1 if platform p is positioned at g, 0 otherwise (binary)
- $R_{a,p,g,d}$ 1 if attack a is engaged by platform p from position $g \in G_{c_p}$ exercising defense option d, 0 otherwise (binary)

Formulation of JD-MINMAX. We specify a set of dual variables, in square brackets, for each constraint of the inner (maximization) problem in JD-MINMAX. These duals are only defined (and used) for solutions to the linear programming relaxation of the integer linear program that results when **X** and **R** are fixed.

$$Z^* = \min_{(\mathbf{X},\mathbf{R})\in \mathrm{XR}} \left\{ \begin{array}{lr} \max\limits_{\mathbf{Y}} \ \sum\limits_t val_t \sum\limits_{a\in A_t} \mathrm{Pk}_a \Big(1 - \sum\limits_{p,g,d} \mathrm{Pn}_{a,c_p,g,d} R_{a,p,g,d}\Big) Y_a & \text{(A0)} \\ \text{s.t. } \sum\limits_s W_{m,s} \leqslant \overline{mobile}_m \quad \forall m \quad [\alpha_m], & \text{(A1)} \\ \quad -W_{m,s} + \sum\limits_{a\in A_{m,s}} Y_a \leqslant \overline{fixed}_{m,s} \quad \forall m,s \quad [\beta_{m,s}], & \text{(A2)} \\ \quad \sum\limits_{a\in A_t} Y_a \leqslant \overline{missiles}_t \quad \forall t \quad [\gamma_t], & \text{(A3)} \\ \quad \underline{move}_{m,s} \leqslant W_{m,s} \leqslant \overline{move}_{m,s} \quad \text{and integer} & \\ \qquad \forall m,s \quad [-\underline{\pi}_{m,s}, \overline{\pi}_{m,s}], & \text{(A4)} \\ \quad 0 \leqslant Y_a \leqslant 1 \quad \text{and integer } \forall a \quad [\theta_a]. & \text{(A5)} \end{array} \right\}$$

The notation $(\mathbf{X},\mathbf{R}) \in \mathrm{XR}$ denotes all feasible prepositioning and interception plans for the defender. This feasible set is described in detail below.

The attacker's objective (A0) expresses total expected target damage, assuming a cumulative effect across targets, and for multiple missiles striking a single target. Constraints (A1) limit the number of mobile missiles of each type that can be transported to launch sites. Constraints (A2) limit the maximum number of missiles of each type, both mobile and fixed, that can be launched from each launch site. Constraints (A3) limit the number of missiles that can attack each target. Constraints (A4) limit the number of mobile missiles of each type that can be transported to each launch site.

The objective (A0) expresses expected incremental target value damage inflicted as a consequence of each attacking missile. For an area target, such as a city or airfield, such a cumulative damage model is standard (e.g., Eckler and Burr 1972). But a point target might be destroyed by any single attacking missile, and the lack of a joint probability expression for surviving more than one hit means that the attacker can be over-credited with damage value. (This problem disappears if the attacker can launch no more than one missile at any target, which can be enforced through constraints (A3).) We believe that when it comes to weapons of mass destruction carried by TBMs, the damage to an economy and a society will continue to increase as the number of successful missile strikes increases. Thus, the cumulative model of damage is appropriate, although there might be some diminishing returns to an attacker as the number of successful strikes on a target (or in a target area) increases. Appendix A suggests how to modify the objective function for diminishing returns or point targets, should these issues arise.

The defender's actions are limited by $(\mathbf{X}, \mathbf{R}) \in \mathrm{XR}$, where XR is defined by the following set of constraints:

$$\sum_g X_{p,g} \leqslant 1 \quad \forall p, \tag{D1}$$

$$\sum_p X_{p,g} \leqslant 1 \quad \forall g \quad \text{(optional)}, \tag{D2}$$

$$\sum_{p,g,d} R_{a,p,g,d} \leqslant 1 \quad \forall a, \tag{D3}$$

$$\sum_{a,d} salvo_{a,c_p,d,i} R_{a,p,g,d} \leqslant loadout_{p,i} X_{p,g} \quad \forall p, i, g \in G_{c_p}, \tag{D4}$$

$$\sum_{a,g,d} salvo_{a,c_p,d,i} R_{a,p,g,d} \leqslant \overline{shoot}_p \quad \forall p \quad \text{(optional)}, \tag{D5}$$

$$\sum_d R_{a,p,g,d} \leqslant X_{p,g} \quad \forall a, p, g, \tag{D6}$$

$$\text{all } X_{p,g}, R_{a,p,g,d} \in \{0,1\}. \tag{D7}$$

Each constraint (D1) limits a platform to occupy at most one grid position; each constraint (D2) optionally limits a grid position to accommodate at most one platform; each constraint (D3) allows at most one interception of each attack; each constraint (D4) limits the number of interceptor engagements from each positioned platform and grid-point combination; each constraint (D5) optionally limits the total number of interceptors that a platform can shoot in the short period of time that elapses in an exchange; each constraint (D6) permits an engagement only from an occupied platform and grid-point combination; and constraints (D7) require binary decisions. Note that constraints (D3) do not require a response for every attacking missile. Indeed, if defenses are overwhelmed, it may be impossible to intercept every missile launched, and we must allow for this eventuality.

The attacker plans to maximize expected damage, and the defender plans to minimize the attacker's maximum expected damage.

2.3. Solving JD-MINMAX with JD-MILP

Direct solution of a min-max model like JD-MINMAX is impossible with standard software. We could create a specialized decomposition algorithm for solving it, along the lines of Israeli and Wood (2002), but prefer a simpler method if one exists. In this case it does: Although the attacker's decision vector $\mathbf{W}$ is integer and $\mathbf{Y}$ is binary, the constraint matrix involving $\mathbf{W}$ and $\mathbf{Y}$ is totally unimodular and all corresponding right-hand side data are integer. Thus, all solutions to the linear-programming relaxation of the attacker's maximizing problem are intrinsically integer. Therefore, we can simply take the linear-programming relaxation of the inner problem to create an inner maximization that is a linear program. We then use the dual variables defined above, and take the dual of that inner maximization to create a "min-min" problem. This results in a simple, minimizing MILP, which we solve using standard optimization software. The MILP is

JD-MILP

$$\min_{\substack{\boldsymbol{\alpha},\boldsymbol{\beta},\boldsymbol{\gamma},\boldsymbol{\pi},\boldsymbol{\theta},\\ \mathbf{X},\mathbf{R}}} \sum_m \overline{mobile}_m \alpha_m + \sum_{m,s} \overline{fixed}_{m,s} \beta_{m,s} + \sum_t \overline{missiles}_t \gamma_t - \sum_{m,s} \underline{move}_{m,s} \underline{\pi}_{m,s} + \sum_{m,s} \overline{move}_{m,s} \overline{\pi}_{m,s} + \sum_a \theta_a \tag{T0}$$

$$\text{s.t. } \alpha_m - \beta_{m,s} - \underline{\pi}_{m,s} + \overline{\pi}_{m,s} \geqslant 0 \quad \forall m, s, \tag{T1}$$

$$\beta_{m_a,s_a} + \gamma_{t_a} + \theta_a + \sum_{p,g,d} \mathrm{Pk}_a val_{t_a} \mathrm{Pn}_{a,c_p,g,d} R_{a,p,g,d} \geqslant \mathrm{Pk}_a val_{t_a} \quad \forall a, \tag{T2}$$

$$\sum_g X_{p,g} \leqslant 1 \quad \forall p, \tag{T3}$$

$$\sum_p X_{p,g} \leqslant 1 \quad \forall g, \tag{T4}$$

$$\sum_{p,g,d} R_{a,p,g,d} \leqslant 1 \quad \forall a, \tag{T5}$$

$$\sum_{a,d} salvo_{a,c_p,d,i} R_{a,p,g,d} - loadout_{p,i} X_{p,g} \leqslant 0 \quad \forall p, i, g \in G_{c_p}, \tag{T6}$$

$$\sum_{a,g,d} salvo_{a,c_p,d,i} R_{a,p,g,d} \leqslant \overline{shoot}_p \quad \forall p, \tag{T7}$$

$$\sum_d R_{a,p,g,d} \leqslant X_{p,g} \quad \forall a, p, g, \tag{T8}$$

$$\text{all } \alpha_m, \beta_{m,s}, \gamma_a, \underline{\pi}_{m,s}, \overline{\pi}_{m,s} \geqslant 0, \quad \text{all } X_{p,g}, R_{a,p,g,d} \in \{0,1\}. \tag{T9}$$

The solution of JD-MILP yields an optimal defense prepositioning plan $\mathbf{X}^*$ and interceptor-commitment plan $\mathbf{R}^*$. We recover the associated, optimal mobile-missile transport plan $\mathbf{W}^*$ and attack plan $\mathbf{Y}^*$ by fixing $\mathbf{X} = \mathbf{X}^*$ and $\mathbf{R} = \mathbf{R}^*$ in JD-MINMAX, and solving the linear program that results.

JD-MILP can be embellished with additional features as long as the modifications can be expressed linearly in $(\mathbf{X}, \mathbf{R}) \in$ XR, and the embellishments that modify the attacker's constraints (A1)–(A5) do not destroy their total unimodularity. (If maintaining total unimodularity in the attacker's optimization is too restrictive, more general solution methods apply, as mentioned above.)

2.4. Variants of JOINT DEFENDER to Assess the Value of Flexibility

By tightening or relaxing constraints (D1)–(D7) on the defender and solving the resulting versions of JD-MILP, we can assess the value of flexibility, or the lack thereof, to the defender. For instance, a commander might not currently be able to place an AEGIS platform in a set of positions G' that is threatened by the adversary's coastal defenses. The commander could solve JD-MILP with and without G' included in G, and determine whether or not it is worthwhile to neutralize those coastal defenses. (This comparison we envisage still assumes transparency between the sides, and that the attacker will know that his defenses have been neutralized and that the previously inaccessible positions are now available to the defender.)

2.5. The Value of Secrecy

JD-MILP's assumption of complete transparency between attacker and defender can lead to unappealing (but logical) outcomes. Suppose, for example, that a defender has two assets to defend, has two interceptors for that defense, and each interceptor has a Pn of 1. Further, assume that he is opposed by an attacker who has two missiles that can strike either target (asset), each with a Pk of 1. Because the attacker can see the defender's preparations, he will destroy at least one target—with probability 1. This may be unappealing because, in the familiar setting of a two-person zero-sum game with randomized strategies, the defender can have a positive probability of losing neither of his assets. Of course, the game-theoretic setting requires opacity, i.e., each opponent must hide his intentions from the other. But, completely hiding missile launch sites and interceptor platforms such as ships is impossible.

On the other hand, both attacker and defender probably do not have complete knowledge of their opponent's plans. To handle this issue, we can modify JD-MINMAX, and JD-MILP correspondingly, to represent situations in which *some* of the defender's assets are hidden from the attacker, and/or *some* launch sites or missiles are hidden from the defender. We refer to the defender being able to conceal part of his decision, fooling the attacker into basing his strategy on partial, or bogus data, and then taking advantage of that deception. This section discusses this case and its converse, where the attacker can conceal some information from the defender.

The Value of Defender Secrecy. The following procedure will evaluate the advantage the defender can gain by hiding the existence of a subset of his platforms from the attacker:

(1) Solve the standard version of JD-MILP to determine total expected damage Z^* under the assumption that the defender's platforms are all visible to the attacker.

(2) Remove platforms whose existence the defender can hide; the attacker knows nothing whatsoever about these platforms.

(3) Solve this modified version of JD-MILP for the visible defense strategy $(\mathbf{X}^*, \mathbf{R}^*)$, and recover the attacker's optimal strategy $\mathbf{Y}^*$.

(4) Fix the "visible-defense strategy" $(\mathbf{X}^*, \mathbf{R}^*)$ and the unsuspecting attacker's strategy $\mathbf{Y}^*$ in JD-MINMAX and solve the defender's minimization to determine the optimal strategy for the hidden platforms, and the total expected damage Z^{**}, given the attacker's obviously suboptimal strategy.

(5) Because of the attacker's suboptimal strategy, $Z^{**} \leqslant Z^*$, so that $Z^* - Z^{**}$ may be viewed as "the value of secrecy" to the defender.

If we hide all defending platforms, and use this procedure, we are estimating the "value of a total surprise defense." (This emulates current planning tools.)

The Value of Attacker Secrecy. Suppose that the defender has gained enough information to be able to, in essence, fix all the variables $W_{m,s}$ in JD-MINMAX. That is, he knows the exact location of every missile the attacker possesses. Both sides solve the resulting restricted version of JD-MILP and determine the total expected damage Z^{**}; let $\mathbf{X}^{**}$ represent the defender's optimal pre-positioning plan for this situation. Now, if the attacker can transport his missiles from site to site without being observed, and do this optimally, he may be able to increase expected damage, because the defender has been fooled and will use his original, now suboptimal, pre-positioning plan $\mathbf{X}^{**}$. So, the attacker solves JD-MILP with $\mathbf{X}$ fixed at $\mathbf{X}^{**}$ (for simplicity, we allow the defender to reoptimize interceptor commitments $\mathbf{R}$), determines an optimal "missile-transport plan" $\mathbf{W}^*$, and optimal attack plan $\mathbf{Y}^*$ with objective value Z^*. Clearly, $Z^{**} \leqslant Z^*$, and the difference $Z^* - Z^{**}$ represents the value of secrecy to the attacker.

Suppose that the attacker can fool the defender into thinking he will never launch his missiles, or that he has none at all. In that case, $(\mathbf{X}^{**}, \mathbf{R}^{**}) = (\mathbf{0}, \mathbf{0})$, i.e., no defense, is a reasonable response from the defender. If we fix $(\mathbf{X}, \mathbf{R}) = (\mathbf{0}, \mathbf{0})$ in JD-MINMAX and solve the resulting linear program to obtain Z^*, we can determine the "value of a total surprise attack" by comparing Z^* to the optimal objective from JD-MILP for a baseline, nonsurprise scenario.

3. Case Study: North Korea, Circa 2010

We have developed two North Korean scenarios, circa 2010, which specify a North Korean arsenal of ballistic missiles and launch sites, a U.S. contingent of ballistic missile defense platforms, and a list of targets with associated

target values. We use these scenarios, and variants, to demonstrate JOINT DEFENDER. In the basic scenario we put each North Korean missile at a specific launch site, but we also report cases in which the missiles are transportable.

When no confusion results, we use the term JOINT DEFENDER to mean JD-MINMAX, or JD-MILP, or the full decision-support system that incorporates these models, prepares data for problem generation, solves the problem, and returns solutions in accessible format.

3.1. The Attacker's Launch Sites

The attacker's hypothetical missile launch sites are based on known North Korean missile facilities and bases taken from unclassified sources (Federation of American Scientists 2003). Table 1 lists these sites, and Figure 4 shows their approximate locations.

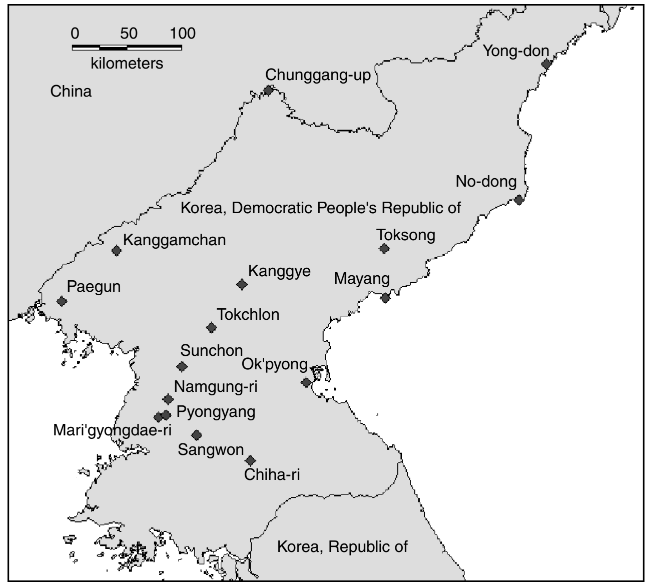
3.2. Attacker Missiles

Table 2 displays the selection of missiles from the North Korean inventory we model in this scenario, along with their approximate minimum and maximum ranges. These ranges have been compiled from unclassified sources (e.g., Jane's 2003d). We assume that each missile hits and destroys its assigned target with perfect reliability, i.e., $\mathrm{Pk}_a = 1.0$ for any a, if the missile is not intercepted. This expresses the worst-case situation.

3.3. Targets on a Defended Asset List (DAL)

Table 3 displays the defended asset list (DAL) and target values for all scenarios, and Figure 5 displays target locations on an area map. We generate target values for the

Figure 4. Each diamond indicates a North Korean launch site.



DAL based upon a subjective assessment of the four factors currently used in air-defense planning: *criticality, vulnerability, reconstitutability*, and *threat* (e.g., Department of the Army Field Manuals FM 3-01.11 2000a and FM 44-100 2000b).

Target t's criticality c_t judges the degree to which an asset is essential to the defender. A high value indicates that the asset is extremely critical, and a low value indicates otherwise.

Table 1. North Korean launch sites (after Federation of American Scientists 2003). These comprise current North Korean missile-production facilities and missile bases, and are used in our scenarios as potential launch sites. For fixed-launch-site scenarios, the maximum number and type of each North Korean ballistic missile is shown for each launch site. When we permit transporting mobile missiles, this same inventory of Scud-B, Scud-C, and No-Dong missiles is mobile.

			Missile types				
Launch sites	Latitude (N)	Longitude (E)	Scud-B	Scud-C	No-Dong	Taep'o-Dong I	Taep'o-Dong II
Chiha-ri	38°37′	126°41′	15	20	10		
Chunggang-up	41°46′	126°53′		10	10		
Kanggamchan	40°24′	125°12′		15	10		
Kanggye	40°07′	126°35′		15	10		
Mari'gyongdae-ri	38°59′	125°40′	10	20	10		
Mayang	40°00′	128°11′		15	20		
Namgung-ri	39°08′	125°46′	5	15	2	1	1
No-dong	40°50′	129°40′		5	15	1	1
Ok'pyong	39°17′	127°18′	15	15	10		
Paegun	39°58′	124°35′		15	10		
Pyongyang	39°00′	125°45′	15	15	10		
Sangwon	38°50′	126°05′	15	20	10		
Sunchon	39°25′	125°55′	5	15	10		
Tokch'on	39°45′	126°15′	5	15	15		
Toksong	40°25′	128°10′	5	15	15		
Yong-don	41°59′	129°58′			20	1	1

Table 2. North Korean ballistic missile types with their range limits. The Scud-B, Scud-C, and No-Dong missiles are operational today; the intercontinental Taep'o-Dongs are in development.

	Range (km)	
Missile	Minimum	Maximum
Scud-B	40	330
Scud-C	40	700
No-Dong	1,350	1,500
Taep'o-Dong I	2,200	2,900
Taep'o-Dong II	3,500	4,300

Vulnerability v_t represents the degree to which a target is susceptible to an air or missile attack or is vulnerable to surveillance. A high value indicates that the target is extremely vulnerable, i.e., unprotected and in the open; a low value indicates otherwise.

Reconstitutability r_t assesses the degree to which the target can recover from inflicted damage, and incorporates time, the need or lack of need for special repair equipment, and the amount of manpower required to resume normal operation. A high value indicates that the target would need considerable time, equipment and/or manpower to return to normal operation following an attack; a low value indicates otherwise.

Threat h_t subjectively estimates the probability of a target being attacked. A high value indicates that it is nearly certain that the enemy will attack this target.

We combine these factors through

$$val_t = \ln(c_t \times v_t \times r_t \times h_t) + 1,$$

where all c_t, v_t, r_t, and h_t range from about 1 to about 10. The natural log function (ln) is chosen somewhat arbitrarily so that val_t also ranges from about 1 to 10. Our definition of target value can be replaced, but any alternative should address these four important components.

Initially, we allow a defended target to be attacked at most once. We want results that are easy to visualize, so we present point targets, easily located on a map.

Table 4. Ranges for each defender interceptor.

Interceptor	Maximum range (km)
THAAD	250
PAC-2	160
PAC-2GEM	160
PAC-3	70
SM2 block III variants	120
SM3	1,200

3.4. Defensive Platforms

To evaluate the defender's 2010 defense plan, we assume that two AEGIS cruisers are deployed, each with 10 SM3 and 20 SM2 interceptors, along with one AEGIS destroyer with 20 SM2 interceptors. Each AEGIS ship has been configured for ballistic-missile defense and deploys as an independent entity.

The defender also has two land-based defensive assets. He can use one PATRIOT battery, which consists of eight mobile launchers (and support vehicles), each loaded with four PAC-3 missiles, two PAC-2 GEM missiles, and one PAC-2 missile. And, he can use one THAAD battery whose salient features comprise a mobile launcher and 10 interceptors.

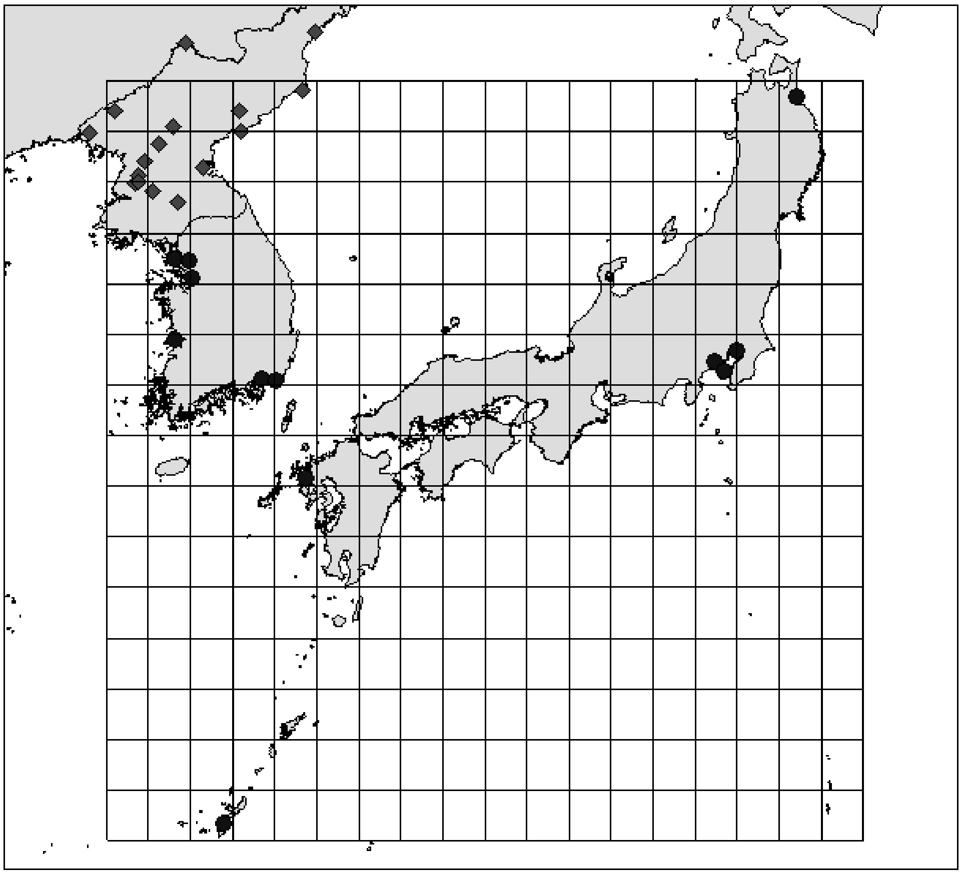
3.5. Interceptor Ranges

Table 4 specifies the maximum range of the various interceptors used by defense platforms in our scenario. Ranges are gleaned from the open literature (Jane's 2003b, c, e).

Table 3. Targets on a defended asset list (DAL). Targets are on this list because of their political or military significance and are spread out over South Korea, the main islands of Japan, and Okinawa. Each target is assigned four scores, reflecting criticality, vulnerability, reconstitutability, and threat. For example, Seoul has (c, v, r, h) values of $(4, 8, 5, 9)$, which result in a target value of $\ln(4 \times 8 \times 5 \times 9) + 1 = 8.3$. The example values shown here are completely arbitrary.

Target	Latitude (N)	Longitude (E)	c	v	r	h	val
Atsugi, JP	35°27′	139°27′	4	7	6	5	7.7
Misawa, JP	40°42′	141°25′	8	5	7	5	8.2
Okinawa, JP	26°20′	127°47′	7	7	8	3	8.1
Sasebo, JP	33°09′	129°44′	7	8	7	7	8.9
Tokyo, JP	35°41′	140°00′	4	9	4	7	7.9
Yokosuka, JP	35°17′	139°40′	8	8	7	7	9.1
Chinhae, ROK	35°08′	128°41′	7	7	7	8	8.9
Inchon, ROK	37°29′	126°38′	3	6	5	4	6.9
Kunsan, ROK	35°54′	126°37′	10	7	9	10	9.7
Osan AB, ROK	37°06′	127°02′	10	8	9	10	9.9
Pusan, ROK	35°06′	129°02′	8	7	8	10	9.4
Seoul, ROK	37°27′	126°57′	4	8	5	9	8.3

Figure 5. Candidate platform positions for the defender. Each circle in Japan and South Korea represents a target; each diamond in North Korea represents an attacker launch site; sea-based platforms can be located at any grid point at sea; land-based platforms can be based at any grid point on land excluding those in North Korea and China (the upper left-hand corner); land-based platforms can also be collocated with targets. For simplicity, grid points are placed at each integer value of latitude and longitude. In reality, the defender's candidate locations could be specified with much greater freedom.



3.6. Interceptor Effectiveness: Probability of Negation (Pn)

For simplicity, these test cases assume "reasonable engagement conditions," which means that a nonterminal interceptor is within range of an attacking missile's trajectory, or a terminal defender is, effectively, collocated with the target of an attack. When these conditions are met, we set the probability of negation (Pn) for an interceptor to a reasonable but hypothetical value between 0.7 and 0.9, and set it to 0.0 otherwise. Alternatively, JOINT DEFENDER could employ a set of (potentially enormous) tables that provide interceptor effectiveness indexed by interceptor type and engagement geometry as specified by cross-range and down-range proximity, and by the attacking missile's altitude. (With respect to the great circle arc connecting the launch site to the target of the attacking missile, the "cross-range" proximity is the distance from the defending platform to the closest point of the arc, and the "down-range" proximity is the distance from this closest point to the target.)

We derive the joint probability that a salvo of interceptors negates an attacker's missile from the negation probabilities of the missiles that comprise a salvo. In this paper, we assume independence between interceptors in a salvo, but JOINT DEFENDER does not require this.

3.7. Candidate Defender Positions

We discretize candidate defender positions into a latitude and longitude grid with increments of one degree, about 60 nautical miles; see Figure 5. This discretization yields 304 candidate grid locations for pre-positioning interceptor platforms, although geography precludes certain classes from being assigned to certain positions. Terminal defensive platforms can also collocate with targets. In addition to obvious restrictions to locate land units on land, and to position ships at sea, we have defined an optional, restricted set of sea positions that are at least 100 nautical miles from the North Korean coast. This puts the ships outside of the 60 nautical-mile range of North Korea's shore-based HY-1

Silkworm and HY-2 Seersucker anti-ship missiles (Federation of American Scientists 2004, Department of the Army 1999).

Our approach does not depend on the structure of the set of candidate locations; we only require that the list be finite. In a real scenario the area commander might nominate a list of candidate positions for JOINT DEFENDER to evaluate based on his expert knowledge of the theater and the capabilities of the platforms under his command.

3.8. Scenario Variants

We develop a sequence of scenario variants to illustrate how the defender or attacker can evaluate flexibility in their strategy.

We depict postures for the defender in which:

(D1) The defender does nothing. This establishes a *worst-case baseline for any surprise attack*.

(D2) The defender hides nothing, including his commitments to intercept each potential attacking missile. This is *the completely flexible and transparent case where attacker and defender each have complete knowledge of each other's plans*.

(D3) The defender lets his platform locations be seen, keeps his ships out of range of shore-based anti-ship missiles, and hides his interceptor commitments.

(D4) The defender lets his platform locations be seen, suppresses shore-based threats to his ships as necessary, positions his ships as close to shore as he pleases, and hides his interceptor commitments.

(D5) The defender hides the positions of his ships, does not hide the positions of his ground-based interceptors, but does hide all interceptor commitments.

(D6) The defender hides everything so that the defense is a complete surprise to the attacker. This establishes a *best-case baseline for whatever the attacker decides to do*. (This is the case assumed by current TBMD planning tools.)

Posture D2 is our baseline. JD-MINMAX represents this "perfectly transparent" case by allowing the attacker to see both the defender's pre-positioning decisions $\mathbf{X}$, as well as his interceptor commitments $\mathbf{R}$. This constitutes a restriction of the defender's capabilities in a real engagement, in which (a) the attacker would observe only the defender's pre-positioning; (b) he would plan and launch an attack given that information; (c) the defender would observe the attack; (d) and only then, after knowing the details of the attack, would the defender need to commit (allocate) interceptors from his pre-positioned platforms. The unrestricted model would be, however, significantly harder to solve than JD-MINMAX.

In postures D3 through D5, we relax our baseline posture to model the case in which the attacker has information about the locations of some or all of the defensive platforms but does not know specific interceptor commitments. Exact solution of this model would also be difficult, so we approximate this situation by (a) solving JD-MINMAX as if the defender were using posture D2 (revealing both position and intercept commitments), (b) fixing the resulting attack plan, and then (c) letting the defender re-allocate his interceptors to engage the attacking missiles more effectively.

In addition, we evaluate postures D7 through D10 under the same assumptions as D4, with one defending platform of each type omitted from the theater. (Specifically, D7 omits CG48, D8 omits DDG68, D9 omits Pbat1, and D10 omits Tbat1; see platform names in Table 6.)

We depict postures for the attacker in which:

(A1) The attacker must use a fixed launch site for each missile, or

(A2) The attacker transports mobile missiles in secret to any launch site he chooses, while deceiving the defender into expecting fixed launch sites.

4. Results and Analysis

We generate JD-MILP using the general algebraic modeling system (GAMS) (Brooke et al. 1998) and solve it with CPLEX 9.0 (ILOG 2003) on a 2 GHz laptop computer operating under Windows XP (Microsoft Corporation 2004). The largest models encountered in analyzing the North Korean cases have, after filtering and presolve reductions, about 120,000 binary variables, 250 continuous variables, and 6,000 constraints. In our experience, posture D6 (a surprise defense, the case assumed by current TBMD planning tools) can be solved optimally in a few seconds. A good solution to the more nuanced cases, such as posture D2 (where attacker and defender have complete knowledge of each other), is discovered within a minute or two, although proving near optimality with a 1% relative tolerance can take a half hour or more.

4.1. Multimissile Attack with No Defense

Table 5 lists an optimal multimissile attack that launches a single missile from each fixed launch site at an undefended target. This produces a total expected damage of 103.0 (each attacking missile is assumed to hit its target). In this "posture," D1-A1, the defender does nothing and the attacker uses fixed launch sites.

Figure 6 illustrates the tracks that attacking missiles would follow in this scenario.

4.2. An Optimal Defense Plan

Assuming that the attacker does not observe defensive preparations, the defender positions his assets to intercept an optimal, theaterwide attack (this is posture D6-A1, with positions shown in Table 6), and reduces expected damage from 103.0 to 1.0. The defender knows in advance about all optimal attack opportunities, so he positions his defensive platforms and engages the attacker's missiles with interceptors having high probabilities of negation. The

Table 5. An optimal, theaterwide undefended attack plan. There are no defensive interceptions at all. Each target on the defended asset list is attacked with a single missile producing a combined expected damage of 103.0. (See map in Figure 6.)

Target	Launch site	Missile type	Expected damage
Atsugi, JP	Kanggamchan	No-Dong	7.7
Misawa, JP	Kanggamchan	No-Dong	8.2
Okinawa, JP	Chiha-ri	No-Dong	8.1
Sasebo, JP	Chiha-ri	Scud-C	8.9
Tokyo, JP	Kanggamchan	No-Dong	7.9
Yokosuka, JP	Kanggamchan	No-Dong	9.1
Chinhae, ROK	Chiha-ri	Scud-C	8.9
Inchon, ROK	Chiha-ri	Scud-B	6.9
Kunsan, ROK	Chiha-ri	Scud-B	9.7
Osan AB, ROK	Chiha-ri	Scud-B	9.9
Pusan, ROK	Chiha-ri	Scud-C	9.4
Seoul, ROK	Chiha-ri	Scud-B	8.3

Figure 6. Map of an optimal, theaterwide undefended attack plan. Maximal attacks are shown with at most one missile aimed at each target and with no interceptions. Maximum expected damage is 103.0. (See data in Table 5.)

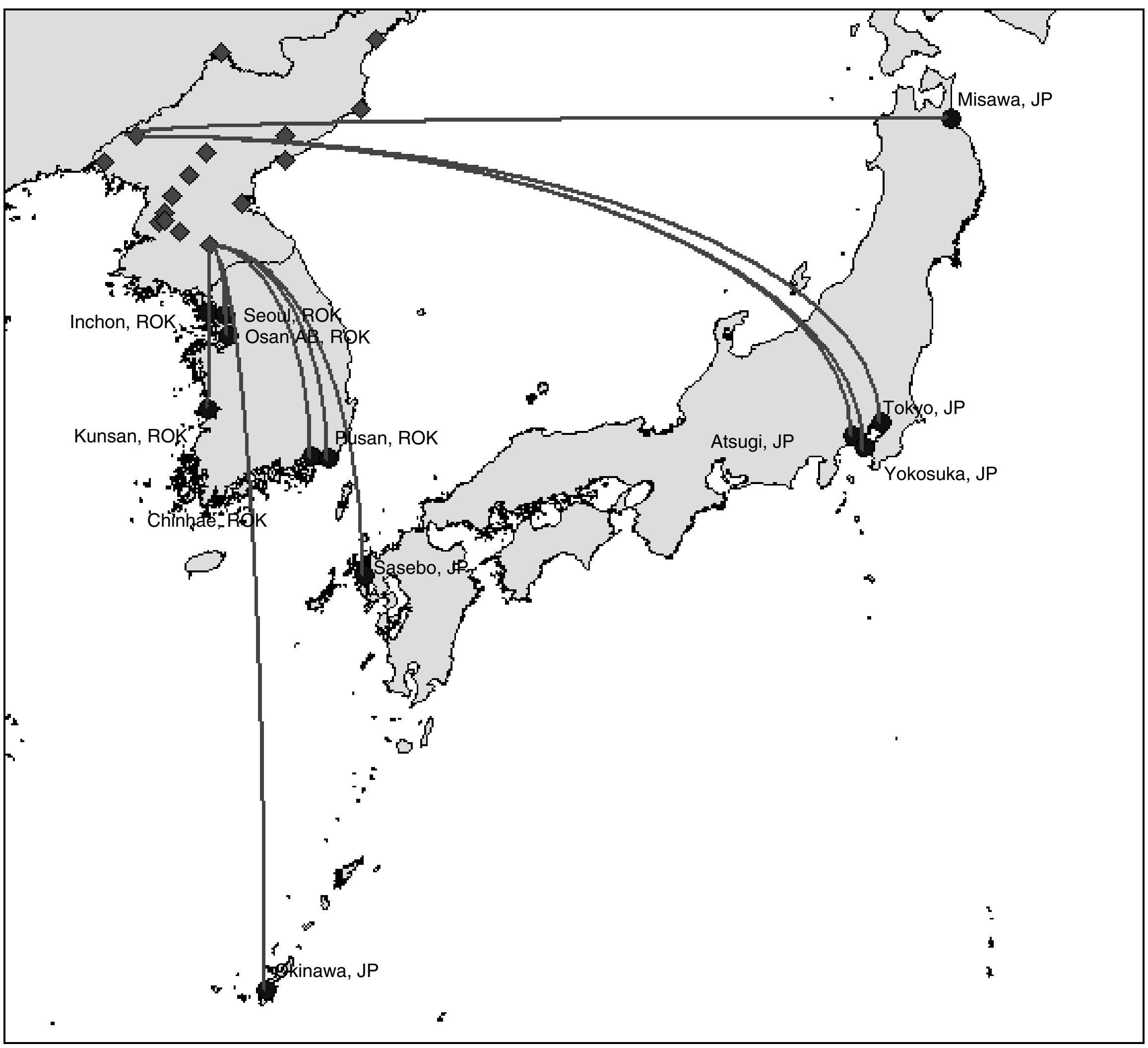


Table 6. Optimal defender positions maintaining defense secrecy against an optimal theaterwide attack. From these (hidden) positions, defending platforms intercept all attacking missiles, but do not necessarily destroy every missile intercepted. The maximum expected damage is reduced to 1.0, or about one-tenth of an attacking missile leaking through.

Defender class	Platform	Latitude (N)	Longitude (E)
AEGIS CG	CG47	35°00′	130°00′
AEGIS CG	CG48	34°00′	129°00′
AEGIS DDG	DDG68	36°00′	126°00′
PATRIOT	Pbat1	37°06′	127°02′
THAAD	Tbat1	40°42′	141°25′

expected damage, evaluating to 1.0, derives from approximately one-tenth of an attacking missile leaking through over all engagements.

Figure 7 illustrates the defender's positions relative to an optimal attack plan, and the subsequent, optimal engagements of the plan's attacking missiles.

4.3. Assume Transparency: A Two-Sided Optimization

If each side can observe *everything* the other intends to do, the attacker knows that the defender may commit an interceptor salvo to each candidate missile attack, and shoot it if he launches that missile. The defender knows that the attacker may get some of his missiles through. The defender's objective is to minimize maximum expected damage, given the attacker can see and take advantage of pre-positioned, defensive forces. This is posture D2-A1. The two-sided attack and defense produces an optimal set of interceptor commitments against threatened launches and shots at missiles launched, as well as, perhaps, some launches against which there is no available defense. Here, the expected damage of 22.7 represents loss of undefended targets Inchon and Chinhae, but still represents *an overall reduction in expected damage from a surprise attack of 78 percent*. Inchon and Chinhae are undefended because available interceptors cannot cover all possible attacking missiles.

In posture D4-A1 we model the situation where the attacker observes platform positions for the defender, but the defender keeps his interception decisions secret. Tables 7 and 8, respectively, illustrate such a defense and the attacking missiles engaged, and Figure 8 depicts the missile attacks, defense, and engagements on a map of the theater.

4.4. A More Stressful Scenario Showing How to Evaluate Partial Transparency, Secrecy, Deception, and the Incremental Value of Each Defender Platform

Now consider a more stressful case for the defender that is too cluttered to illustrate on a map of the theater. Suppose that the attacker can launch as many as three missiles of any type from any launch site, and that each target can be attacked as many as 10 times. Posture D3-A1 exhibits 96 attacks, defended by 53 intercepting salvos using 90 interceptors. Total expected target damage is 394.4, with 43 attacking missiles expected to leak through defenses. Maintaining total defender secrecy, D6-A1, reduces total expected damage to 152.2.

Suppose that the defender can keep naval defensive platforms hidden from the attacker, but the attacker can observe all land-based defenses; this is posture D5-A1. The resulting expected damage changes from the upper bound of total transparency toward the lower bound of total defender secrecy; see Figure 9. The difference between the expected damage in the transparent solution and the expected damage of this solution is the value of partial defender secrecy. In practical terms, this value quantifies how an increase in information hiding either through funding, strategy, or a combination of both, will reduce the attacker's ability to inflict damage.

The value of partial defender secrecy is bounded by the difference between the expected damage of the completely transparent solution and the expected damage given complete defender secrecy. In the latter case, the defender knows which individual attacks will occur and hides the existence of all interceptors from the attacker (with resulting value $394.4 - 152.2 = 242.2$).

The attacker may gain some advantage from secretly transporting missiles to alternative launch sites. Defending ships are most affected by this deception. In contrast, the PATRIOT battery provides a terminal defense that is relatively insensitive to an incoming missile's track, which depends on its origin.

We present defensive postures D7 through D10 to assess the value of each defending platform. Here, we assume that posture D4 applies (platforms are seen by the attacker, but engagements are concealed), as one platform of each type is successively removed. Table 9 shows the value of each platform, estimated by comparison with all platforms available. The Patriot battery is valuable in both fixed and mobile attack-missile postures (~90 units of expected damage); the Aegis cruiser is more valuable in the mobile attack posture (~110 units of expected damage), but has low value in

Figure 7. Map of optimal defender positions and engagements when defense secrecy is maintained against an optimal, theaterwide attack. From these (hidden) positions, defending platforms intercept all attacking missiles, but do not necessarily destroy every missile intercepted. The maximum expected damage is reduced to 1.0, which corresponds to the expected damage from one tenth of an attacking missile leaking through the defense.

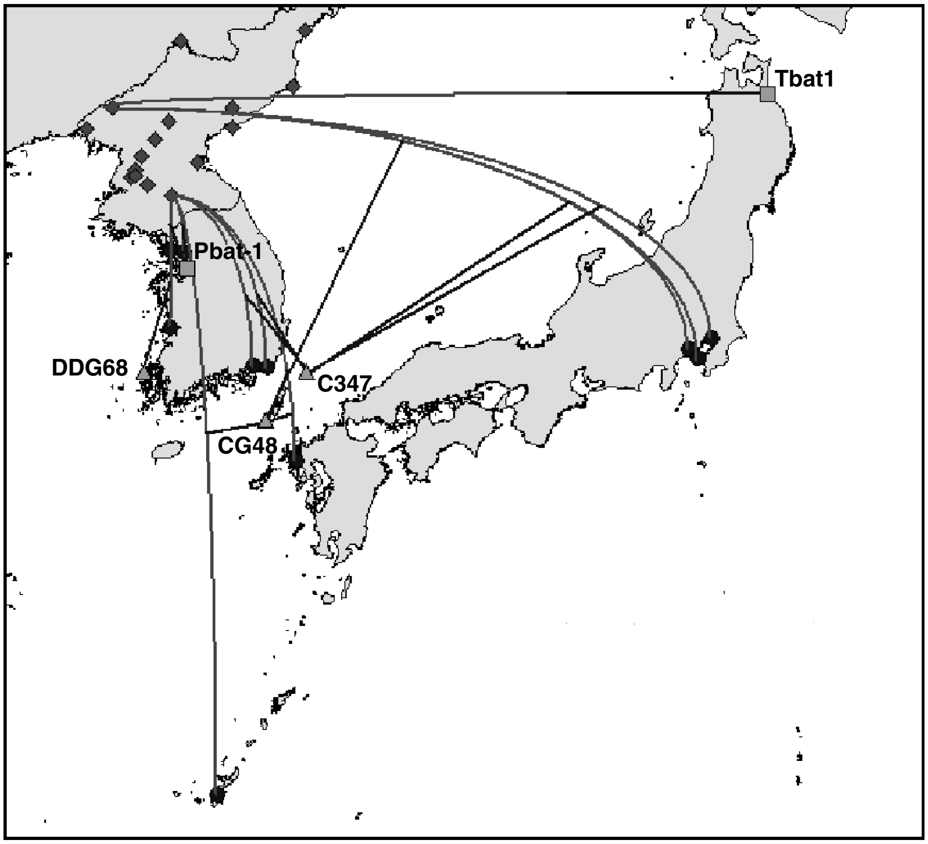


Table 7. Optimal defending platform positions given that these positions are observed by the attacker, but defending interceptor commitments are kept secret. Each defending platform is positioned to minimize the attacker's worst possible attack. The defender, while determining his platform positions, commits interceptors to thwart potential missile attacks that may not actually be launched, but will be intercepted if they are. Once positioned, the defender can intercept any attacking missile he chooses. (See map in Figure 8.)

Defender class	Platform	Latitude (N)	Longitude (E)
AEGIS CG	CG47	34°00′	130°00′
AEGIS CG	CG48	36°00′	126°00′
AEGIS DDG	DDG68	35°00′	130°00′
PATRIOT	Pbat1	37°27′	126°57′
THAAD	Tbat1	40°00′	140°00′

Table 8. An optimal attack plan given that defending platform positions are observed by the attacker, but defending interceptor commitments are kept secret. Each target is attacked with at most one missile. The defender, while determining his platform positions, commits interceptors to thwart potential attacks that may not actually be launched, but will be intercepted if they are. Once positioned, the defender can intercept any attacking missile he chooses. Total defended asset list target value at risk is 103.0 and expected target damage is 1.0. (See map in Figure 8.)

Target	Launch site	Missile type	Salvo option	Salvo Pn	Defender	Expected damage
Atsugi, JP	Kanggamchan	No-Dong	2 SM3	0.99	CG48	0.1
Misawa, JP	Kanggamchan	No-Dong	2 SM3	0.99	CG48	0.1
Okinawa, JP	Chiha-ri	No-Dong	2 SM3	0.99	CG47	0.1
Sasebo, JP	Chiha-ri	Scud-C	2 SM2-III	0.99	CG47	0.1
Tokyo, JP	Kanggamchan	No-Dong	2 SM3	0.99	CG47	0.1
Yokosuka, JP	Kanggamchan	No-Dong	2 SM3	0.99	CG47	0.1
Chinhae, ROK	Chiha-ri	Scud-C	2 SM2-III	0.99	DDG68	0.1
Inchon, ROK	Namgung-ri	Scud-C	2 PAC3	0.99	Pbat1	0.1
Kunsan, ROK	Chiha-ri	Scud-B	2 SM2-III	0.99	CG48	0.1
Osan AB, ROK	Chiha-ri	Scud-B	2 PAC3	0.99	Pbat1	0.1
Pusan, ROK	Chiha-ri	Scud-C	2 SM2-III	0.99	DDG68	0.1
Seoul, ROK	Chiha-ri	Scud-C	2 PAC3	0.99	Pbat1	0.1

Figure 8. Map of an optimal theaterwide attack given that defending platform positions are observed by the attacker, but defending interceptor commitments are kept secret. Each target is attacked with at most one missile. The defender, while determining his platform positions, commits interceptors to thwart potential attacks that may not actually be launched, but will be intercepted if they are. Once positioned, the defender can intercept any attacking missile he chooses. Total defended asset list target value at risk is 103.0 and expected target damage is 1.0. (See data in Tables 7 and 8.)

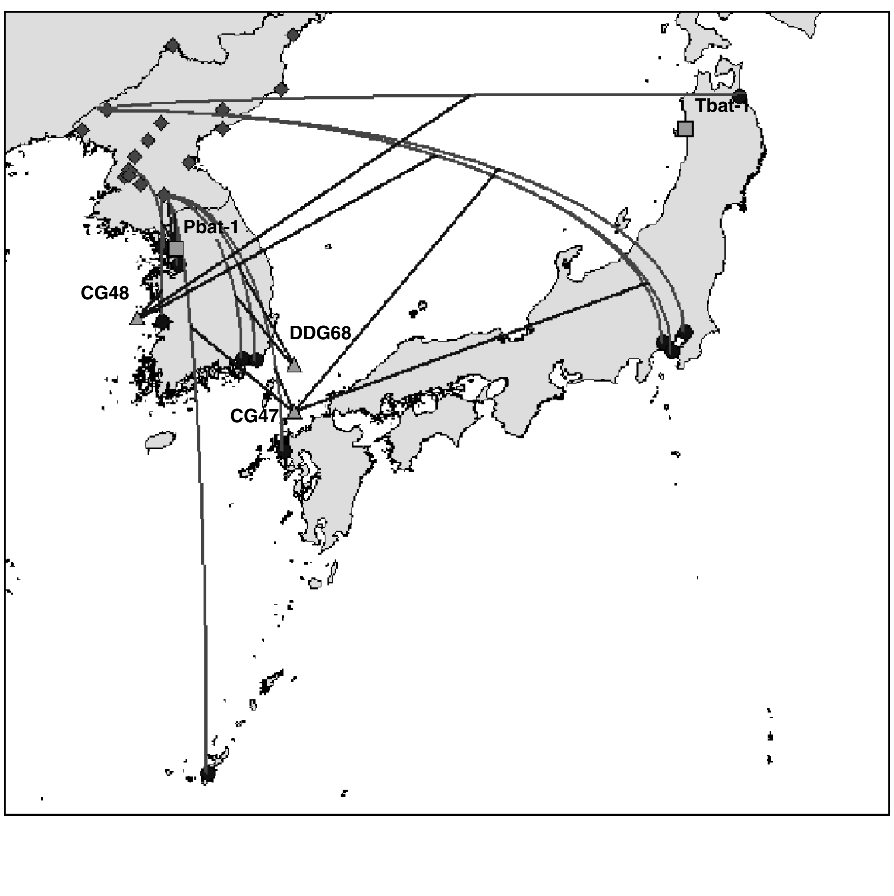


Figure 9. Minimized maximum expected target damage for 20 scenarios mixing defense and attack postures. From D1 to D6, the defender works harder and harder to intercept attacks and conceals more and more information from the attacker, while the attacker either has known, observed launch sites (vertical scale A1) or mobile launch sites hidden from the defender (vertical scale A2). For example, with defending ships hidden from the attacker, posture D5-A1 has expected damage 355.5 with known, fixed attack launch sites, or 394.7 if the attacking missiles can be transported to surprise launch sites D5-A2. Moving defending ships out of range of shore-based anti-ship missiles, D2, does not degrade the defense. The value of secrecy is the positive difference between the expected damage under that level of secrecy and the expected damage in the fully transparent model (e.g., the value of complete defender secrecy is $456.1 - 152.2 = 303.9$).

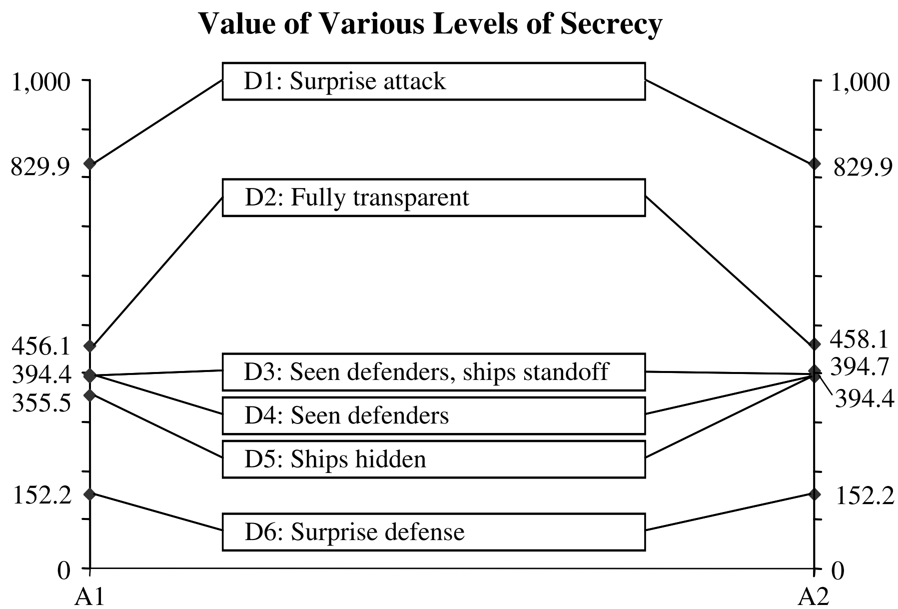


Table 9. Defensive postures D7 through D10 show the effect of removing one of each type of defender platform from the scenario. Posture D4 provides a baseline for comparison. In each scenario, the remaining platforms reposition and resort to defensive salvos using fewer interceptors. For instance, when launch locations are fixed, removing one Aegis cruiser (CG48) results in a moderate increase in expected target damage ($398.5 - 394.4 = 4.1$). But, when launch sites are mobile, removing the same platform results in a dramatic increase in expected target damage ($504.3 - 394.4 = 109.9$).

	A1: Fixed launch sites		A2: Mobile launch sites	
Posture	Expected damage	Increase from D4	Expected damage	Increase from D4
D4: All platforms seen (baseline)	394.4	0.0	394.4	0.0
D7: Remove CG48	398.5	4.1	504.3	109.9
D8: Remove DDG68	432.6	38.2	432.6	38.2
D9: Remove Pbat1	482.6	88.2	482.6	88.2
D10: Remove Tbat1	404.4	10.0	404.4	10.0

the fixed-launch-site attack posture (~4 units of expected damage).

5. Conclusions

We have introduced JOINT DEFENDER, a new optimization-based decision-support tool for pre-positioning theater ballistic missile defense (TBMD) assets, i.e., missile-interceptor platforms. JOINT DEFENDER can model a scenario in which both attacker and defender have knowledge of the other's strategy, it can model no defense at all, and it can model an optimal defense against an attack assuming that the attacker expects no defense. Existing defensive planning tools can only evaluate the last type of scenario, and then, only approximately. JOINT DEFENDER solves such problems exactly on a laptop computer in just a few seconds.

JOINT DEFENDER can also model a more complicated scenario in which a defender first pre-positions his TBMD platforms to protect a set of targets. An attacker observes those defensive positions and, given that information, launches his missiles so as to maximize the total expected value of target damage. The defender can optimize his pre-positioning (and commitments of interceptors to attacking missiles), because he knows the attacker will behave to optimize his own objective function. *The attacker cannot increase expected damage by using any other strategy.*

We develop JOINT DEFENDER as a bilevel integer linear program, but convert it to a standard mixed-integer linear program for solution purposes. We have demonstrated its practicability by solving a number of realistic scenarios involving North Korea, using data gleaned from public sources. We have also explored the "value of secrecy" to both sides of the conflict. JOINT DEFENDER identifies an optimal plan for a typical transparent scenario in a minute or two on a laptop computer, although we find instances that require one half hour to prove optimality.

Two-sided mathematical models of military conflict have been studied since Lanchester (1916). Danskin (1967, p. viii) recounts that, in 1951, the Rand Corporation studied two-sided situations where "one side allocates anti-missile defenses to various cities. The other side observes this allocation and then allocates missiles to those cities." In discussing defense against nuclear strikes, and in addition to using a dual reformulation from max-min to max-max, Owen (1969, p. 491) states: "It is, of course, assumed that the defender must deploy his hardware first; the attacker, in full knowledge of this deployment, will act next." In Appendix B, we establish the relationship between our two-sided model (JD-MINMAX) in JOINT DEFENDER and a game invented by von Stackelberg (1952). These seminal contributions, achieved solely with classical mathematics (i.e., with no computers) but only by asserting many simplifying assumptions (such as continuous activities) still offer prescient insight. We have now discovered how to actually formulate and solve such problems with realistic fidelity.

In contrast to our bilevel optimization, a standard game-theoretic model would assume that the attacker does not observe the positioning of defensive assets before launching his attack, and the defender is unaware of the allocation of attacking missiles to targets (e.g., Matheson 1975), or that either side is unaware of the total number of assets (offensive or defensive) possessed by the adversary. Eckler and Burr (1972) discuss solutions for many versions of such games. Bracken et al. (1987) discuss solutions that are robust with respect to uncertain numbers of attacking assets.

Diehl (2004) provides the contemporary (unclassified) foundation for JOINT DEFENDER, discusses the philosophy of target damage functions, and suggests some alternative solution strategies that we have not pursued here.

JOINT DEFENDER represents a substantial technological advance over existing TBMD planning tools that employ heuristics, or expect the planner to guess at good defense plans, or require supercomputers for implementation. None of these existing tools assumes that the attacker can detect defensive preparations and respond accordingly—*this is a key weakness addressed by JOINT DEFENDER*.

Planners are comfortable with a decision-support tool they can control, so JOINT DEFENDER accepts advice such as "fix this platform in this position," "try this position first," or "we have no advice to offer." Similarly, JOINT DEFENDER accepts, but does not require, other advice on the details of an interception plan, including "evaluate this exact plan."

The Joint Task Force's area air defense commander and regional air defense commander can use JOINT DEFENDER for initial defense planning and assessment, and for assessing the value of hiding information from the attacker. JOINT DEFENDER can also provide insight to TBMD program officers in Washington, DC. For instance, it can evaluate trade-offs between investing in a few, highly effective but expensive interceptors or in larger numbers of relatively inexpensive, but less effective interceptors.

JOINT DEFENDER has been presented to Naval Warfare Development Command (NWDC), to the United States Strategic Command program and requirements staff, and has undergone additional proof testing with a number of scenarios of interest to these organizations. The NWDC's air defense department head, Captain Garry Holmstrom, USN (ret), has stated, "This project has produced a most promising solution to the Joint as well as the Navy's problem of BMD asset allocation, at minimal development and fielding cost." JOINT DEFENDER is now under further development for NWDC in preparation for further testing and future integration into the TBMD planning environment, and for potential use in the Global Command and Control System-Maritime and/or the Area Air Defense Command System-lite.

6. Epilogue

On or about 1 October 2004, the USS Curtis Wilbur, a destroyer of the U.S. 7th Fleet, began patrolling the Sea of

Japan as our first step in building a missile shield for the United States and its allies (*Army Times* 2004).

Appendix A. Variations on JD-MINMAX'S Objective Function

The objective function (A0) models area targets that can be damaged more than once, but not point targets that can be destroyed just once (e.g., Eckler and Burr 1972). If we partition the set of targets T into area and point targets, i.e., $T = \{T_{\text{area}}, T_{\text{point}}\}$, the following objective function models the situation more accurately:

$$\min_{(\mathbf{X},\mathbf{R})\in XR} \max_{\mathbf{Y}} \sum_{t\in T_{\text{area}}} val_t \cdot \sum_{a\in A_t} \left(\text{Pk}_a \left[1 - \sum_{p,g,d} \text{Pn}_{a,c_p,g,d} R_{a,p,g,d}\right]\right) Y_a + \sum_{t\in T_{\text{point}}} val_t \left(1 - \prod_{a\in A_t} \left(1 - \text{Pk}_a \cdot \left[1 - \sum_{p,g,d} \text{Pn}_{a,c_p,g,d} R_{a,p,g,d}\right]\right)^{Y_a}\right). \quad (\text{A0}')$$

But, our linear-programming subproblem for the attacker no longer suffices.

On the other hand, one can argue that, even in the case of area targets, the damage resulting from multiple successful strikes is not additive. For instance, the economy and welfare of a city might suffer almost as much from a single successful missile strike as from two. Ignoring variations in weapon types for simplicity, a sensible modeling technique makes the expected value of a set of successful strikes on a target a concave function of the expected number of such strikes. (Indeed, the simple point-target model fits this description.) We can accommodate this by subtracting larger and larger fractions of expected target value as the number of attacking missiles increases:

$$\min_{(\mathbf{X},\mathbf{R})\in XR} \max_{\mathbf{Y}} \sum_{t\in T_{\text{area}}} val_t \cdot \left[\sum_{a\in A_t} \left(\text{Pk}_a \left[1 - \sum_{p,g,d} \text{Pn}_{a,c_p,g,d} R_{a,p,g,d}\right]\right) Y_a - \sum_{k=1}^{\overline{missiles}_t} \text{Pk}_a f_{tk} Y'_{tk}\right], \quad (\text{A0}'')$$

where (a) Y'_{tk} is 1 if the number of missiles targeted at t is greater than or equal to k and is 0 otherwise, and (b) f_{tk}, $0 \leqslant f_{tk} < 1$ for all t and k, is an increasing function of k; this takes into account some approximation of the probability that each attacking missile is thwarted. The inner subproblem remains totally unimodular because we simply replace constraints (A2) with

$$\sum_{a\in A_t} Y_a - \sum_{k=1}^{\overline{missiles}_t} Y'_{tk} \leqslant 0 \quad \forall\, t,$$

$$0 \leqslant Y_{tk} \leqslant 1 \quad \forall\, t, k.$$

Thus, the inner subproblem remains a linear program with a dual formulation, and a mixed-integer linear program formulation for the overall problem can be created.

If it becomes imperative to model point targets with an objective akin to (A0′), the linear objective of (A0) can be maintained by letting Y_a represent a multiweapon strike and adjusting k_a accordingly. Unfortunately, the constraints necessary to enforce this would result in a subproblem that is not totally unimodular. Brown et al. (2004) (see also the discussion on trilevel defense models in Israeli and Wood 2002) deal successfully with this issue by solving their analog of JD-MINMAX with a specialized decomposition algorithm. That technology also applies to our problem, at least in theory.

Appendix B. Stackelberg Games

The models in JOINT DEFENDER's basic model comprises an instance of a *Stackelberg game* (von Stackelberg 1952; see Luo et al. 1996, pp. 11–15 for an overview), which we represent as a *bilevel integer linear program* (e.g., Moore and Bard 1990). The bilevel program converts to a standard MILP for solution purposes.

The key ingredients of any Stackelberg game are a *leader* (our defender) and a *follower* (our attacker). The basic ("one-play") version of the game we use has two phases: (a) The leader carries out a set of actions to coerce behavior from the follower, and (b) the follower observes the leader's actions and how they have affected his ability to respond and/or the value of responding, and reacts by optimizing his own objective. The leader has an objective, too, which is based on the costs of his own actions and his evaluation of the follower's responses. Because the leader understands and models the follower's optimizing behavior precisely, he can and does optimize his own objective by coercing the follower appropriately. If the follower suboptimizes for some reason, or if the leader has assumed that the follower has more flexibility or ability than he actually does, the leader can guarantee the actual outcome will be no worse than one predicted by the game.

The leader's and follower's objectives in a Stackelberg game need not be diametrically opposed, but are in our TBMD problem. Our attacker (follower) wishes to maximize total expected damage while our defender (leader) wishes to minimize that maximum. Our defender's actions are constrained by the physical limits on deploying TBMD platforms and on the physical limits of his interceptors. Our attacker's actions of firing TBMs are not constrained by our defender's actions, but the values associated with firing TBMs are. Our defender's actions affect the success probabilities for our attacker's missiles and thereby the objective of total expected damage.

References

Army Times. 2004. Destroyer patrol is first step in defense against missile attacks. Associated Press, October 1. Retrieved September 8, 2005. http://www.armytimes.com/story.php?f=1-292925-382918.php.

Bracken, J., P. S. Brooks, J. E. Falk. 1987. Robust preallocated preferential defense. *Naval Res. Logist.* **34** 1–22.

Brooke, A., D. Kendrick, A. Meeraus, R. Raman. 1998. *GAMS: A User's Guide*. GAMS Development Corporation, Washington, DC.

Brown, G. G., W. M. Carlyle, R. C. Harney, E. M. Skroch, R. K. Wood. 2004. Interdicting a nuclear weapons project (in review).

Central Intelligence Agency. 2001. *Foreign Missile Developments and the Ballistic Missile Threat Through 2015*. Unclassified summary of the National Intelligence Estimate produced by the National Intelligence Council. Retrieved May 28, 2004 from http://www.cia.gov/nic/special_missilethreat2001.html.

Danskin, J. M. 1967. *The Theory of Max-Min*. Springer-Verlag, New York.

Department of Defense. 2004. *Fiscal 2005 Department of Defense Budget Release*, February 2. Retrieved March 19, 2004 from http://east.defenselink.mil/releases/2004/nr20040202-0301.html.

Department of the Army. 1999. *FY 99 Air and Missile Defense Master Plan (1999), Chapter 2—Threat*. Retrieved May 24, 2004 from http://fas.org/spp/starwars/docops/amd99.

Department of the Army. 2000a. *Field Manual FM 3-01.11. Appendix A—ADA Employment Principles, Guidelines, and Priorities*. Retrieved May 3, 2004 from http://www.globalsecurity.org/military/library/policy/army/fm/.

Department of the Army. 2000b. *Army Field Manual FM 44-100, Chapter 4. Fundamentals of Army Air and Missile Defense Operations*. Retrieved May 3, 2004 from http://www.globalsecurity.org/military/library/policy/army/fm/.

Diehl, D. D. 2004. How to optimize joint theater ballistic missile defense. Master's thesis, Naval Postgraduate School, Monterey, CA.

Eckler, A. R., S. A. Burr. 1972. *Mathematical Models of Target Coverage and Missile Allocation*. Military Operations Research Society, Alexandria, VA.

Federation of American Scientists. 2003. *DPRK—Special Weapons Facilities*. Retrieved April 13, 2004 from http://www.fas.org/nuke/guide/dprk/facility/index.html.

Federation of American Scientists. 2004. C-201/HY-2/SY-1 CSS-N-2/CSS-C-3/SEERSUCKER. Retrieved May 24, 2004 from http://www.fas.org/man/dod-101/sys/missile/row/c-201.htm.

Fulkerson, D. R., G. C. Harding. 1977. Maximizing the minimum source-sink path subject to a budget constraint. *Math. Programming* **13** 116–118.

Golden, B. 1978. A problem in network interdiction. *Naval Res. Logist. Quart.* **25** 711–713.

ILOG. 2003. ILOG CPLEX 9.0 user's manual. ILOG S.A., Gentilly Cedex, France.

Israeli, E., R. K. Wood. 2002. Shortest-path network interdiction. *Networks* **40** 97–111.

Jane's. 2003a. *C4I Systems, Command Information Systems—Naval, United States*. Retrieved May 19, 2004 from www4.janes.com.

Jane's. 2003b. *Electro-Optic Systems 2002–2003, Naval Systems—Surface-to-Air Missiles, United States*. Retrieved May 19, 2004 from www4.janes.com.

Jane's. 2003c. *Land Base Air Defense, Static and Towed Surface-to-Air Missile Systems, United States*. Retrieved May 19, 2004 from www4.janes.com.

Jane's. 2003d. *Strategic Weapon Systems 38, Offensive Weapons, Korea, North*. Retrieved May 19, 2004 from www4.janes.com.

Jane's. 2003e. *Strategic Weapon Systems 39, Defensive Weapons, United States*. Retrieved May 19, 2004 from www4.janes.com.

Lanchester, F. W. 1916. *Aircraft in Warfare: The Dawn of the Fourth Arm*. Constable and Co., London, UK.

Luo, Z. Q., J. S. Pang, D. Ralph. 1996. *Mathematical Programs with Equilibrium Constraints*. Cambridge University Press, Cambridge, UK.

Matheson, J. D. 1975. Multidimensional preferential strategies. Analytic Services, Inc., report no. SDN 75-3, Arlington, VA, November, National Technical Information Service Number ADA021224/1.

Microsoft Corporation. 2004. Windows XP Home Page. Retrieved May 24, 2004 from http://www.microsoft.com/windowsxp/default.asp.

Moore, J. T., J. F. Bard. 1990. The mixed integer linear bilevel programming problem. *Oper. Res.* **38** 911–921.

Owen, G. 1969. Minimization of fatalities in a nuclear attack model. *Oper. Res.* **17** 489–505.

Phillips, C. 1993. The network inhibition problem. *Proc. 25th Annual ACM Sympos. Theory Comput.* ACM Press, San Diego, CA, 776–785.

Salmeron, J., K. Wood, R. Baldick. 2004. Analysis of electric grid security under terrorist threat. *IEEE Trans. Power Systems* **19** 905–912.

Silicon Graphics Incorporated. 2003. Press release, February 12. Retrieved April 25, 2004 from http://www.sig.com/newsroom/press_releases/2003/february/q2_wins.html.

Skroch, E. 2004. How to optimally interdict a belligerent project to develop a nuclear weapon. Master's thesis, Naval Postgraduate School, Monterey, CA.

Sparta, Inc. 2004. *Commander's Analysis and Planning Simulation* (CAPS). v 8.0 user's manual. Sparta, Inc., San Diego, CA.

von Stackelberg, H. 1952. *The Theory of the Market Economy* (translated from German). William Hodge & Co., London, UK.

Whiteman, P. S. 2000. Improving the effectiveness of single-strike effectiveness for network interdiction. *Military Oper. Res.* **4** 15–30.

Wood, R. K. 1993. Deterministic network interdiction. *Math. Comput. Model.* **17** 1–18.